

### Section 3.3

**Test for Increasing and Decreasing Functions:** Let  $f$  be a function that is continuous on the closed interval  $[a, b]$  and differentiable on the open interval  $(a, b)$ .

1. If  $f'(x) > 0$  for all  $x$  in  $(a, b)$ , then  $f$  is increasing on  $[a, b]$ .
2. If  $f'(x) < 0$  for all  $x$  in  $(a, b)$ , then  $f$  is decreasing on  $[a, b]$ .
3. If  $f'(x) = 0$  for all  $x$  in  $(a, b)$ , then  $f$  is constant on  $[a, b]$ .

**Guidelines for Finding Intervals on Which a Function is Increasing or Decreasing:** Let  $f$  be continuous on the interval  $(a, b)$ . To find the open intervals on which  $f$  is increasing or decreasing, use the following steps.

1. Locate the critical numbers of  $f$  in  $(a, b)$ , and use these numbers to determine test intervals.
2. Determine the sign of  $f'(x)$  at one test value in each of the intervals.
3. Use the test for increasing and decreasing functions (above) to determine whether  $f$  is increasing or decreasing on each interval.

These guidelines are also valid when the interval  $(a, b)$  is replaced by an interval of the form  $(-\infty, b)$ ,  $(a, \infty)$ , or  $(-\infty, \infty)$ .

**The First Derivative Test:** Let  $c$  be a critical number of a function  $f$  that is continuous on an open interval  $I$  containing  $c$ . If  $f$  is differentiable on the interval, except possibly at  $c$ , then  $f(c)$  can be classified as follows.

1. If  $f'(x)$  changes from negative to positive at  $c$ , then  $f$  has a *relative minimum* at  $(c, f(c))$ .
2. If  $f'(x)$  changes from positive to negative at  $c$ , then  $f$  has a *relative maximum* at  $(c, f(c))$ .
3. If  $f'(x)$  is positive on both sides of  $c$  or negative on both sides of  $c$ , then  $f(c)$  is neither a relative maximum nor a relative minimum.

- 1) Find the open intervals on which  $f(x) = \frac{2}{3}x^3 - 2x$  is increasing or decreasing. To do this, fill in the table below (similar to the one in example 1), evaluating  $f'(x)$  at test values in each interval determined by the critical numbers.

<b>Interval</b>			
<b>Test Value</b>			
<b>Sign of <math>f'(x)</math></b>			
<b>Conclusion</b>			

- 2) Find the relative extrema of the function  $f(x) = \tan x - 2x$  in the interval  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ . Fill in the table below to help you.

<b>Interval</b>			
<b>Test Value</b>			
<b>Sign of <math>f'(x)</math></b>			
<b>Conclusion</b>			

- 3) Find the relative extrema of  $f(x) = (x^2 + x - 2)^{2/3}$ . Fill in the table below to help you.

<b>Interval</b>			
<b>Test Value</b>			
<b>Sign of <math>f'(x)</math></b>			
<b>Conclusion</b>			

- 4) Find the relative extrema of  $f(x) = \frac{x^2-3}{x^3}$ . Fill in the table to help you.

<b>Interval</b>				
<b>Test Value</b>				
<b>Sign of <math>f'(x)</math></b>				
<b>Conclusion</b>				

Homework for this section: Read the section and watch the videos/tutorials. Then do these problems in preparation for the quiz: #5, 9, 25, 43, 59, 70